Impact of Atmospheric Refraction on Asr Time

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Abstract: **The influence of atmosphere on the salat times *fajr, maghrib* and *isha* is well-known and thoroughly studied by Muslim researchers. However, its impact on *asr* time, though small, had not been a concern. This paper examines the effect of refraction upon the *asr* time and proposes a simple correction factor into the calculations.**

Keywords: prayer timing; atmospheric refraction; asr; apparent sun angle

1. INTRODUCTION

The five daily prayers are religious obligations for Muslims. Knowing the start & end times of these prayers is essential to fulfill these obligations. These times are defined in the *Qur'an* (*Hud*, 11:114; *Al-Isra*, 17:78; *Ta-Ha*, 20:130) as well as in *Hadith* (*Al-Nasa'i*, 523) [1].

Both *fajr* and *isha* times are expressed in reference to twilight, namely *fajr* begins with the dawn (morning twilight) and *isha* starts after dusk when the evening twilight disappears [2]. Twilight is a direct cause of the atmosphere: Sunlight, scattered by the atmospheric particles, illuminates the lower part of the sky, although the sun is actually well below the horizon. The effect of scattering on red/white twilight has been investigated for a long time in order to estimate a solar depression angle to correspond to the start of dawn and end of dusk.

Similarly, sunrise & sunset are two distinctive events for two other prayer timings: Sunset is the beginning of *maghrib* time, whereas sunrise is the ending time for *fajr* prayer. The existence of the atmosphere also affects the sunset/sunrise times: When the sun just (dis)appears on the horizon, its upper limb is actually (astronomically) below the horizon (Figure 1) because of the refraction of its rays by the atmospheric layers. Moreover, the refraction distorts the shape of the sun when near horizon: It turns into an ellipse, and, in case of high vertical temperature gradient, even into a near-triangle [3].

The difference between the apparent (refracted) and true sun vertical angles, denoted as the refraction angle, is generally accepted as 0.57° at sunrise/sunset, although some observations show rather a range between 0.5° and 0.9°, at sunset being less than at sunrise [4]. A refraction angle of 0.57° causes a drift of 3~4 minutes in sunset/sunrise times for Istanbul (41° latitude). This drift increases to 5 minutes at a latitude of 55°. It should be noted that this drift is positive for sunset (later) and negative for sunrise (earlier).

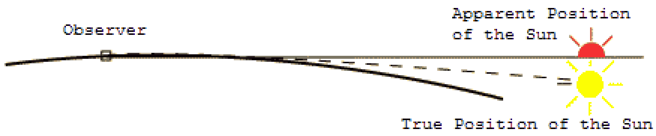

Figure 1 – Refraction at sunrise/sunset

The *dhuhr* prayer starts when the Sun begins to decline after reaching its highest point in the sky at noon (zenith) and its time is independent of atmospheric refraction.

2. CALCULATION of ASR TIME

The remaining prayer is the afternoon (*asr*) prayer. Its beginning time was not explicitly stated by Qur'an, so it was determined by the major schools of Islamic jurisprudence. *Shafi'i*, *Maliki*, *Ja'fari* and *Hanbali* schools determine it as the time when the shadow length of any object reaches its own length plus its shadow length at noon, whereas there is an opinion in the *Hanafi* school which designates that *asr* begins when the object's shadow length is <u>twice</u> its own length plus its shadow length at noon [5].

Noon shadow is the shadow of a vertical pole at the exact solar noon [6], i.e. when the sun has the highest daily altitude. It is also called as the true shadow or declining (*zawal*) shadow and this is the smallest shadow of the object for that day. If the noon shadow of a 1 meter long object is 0.5 meter for example, the *asr* time begins when its shadow becomes 1.5 meter for the first definition and 2.5 meter for the latter (Figure 2). We will denote the first *asr* time as "*asr*-I" and the second as "*asr*-II" from now on.

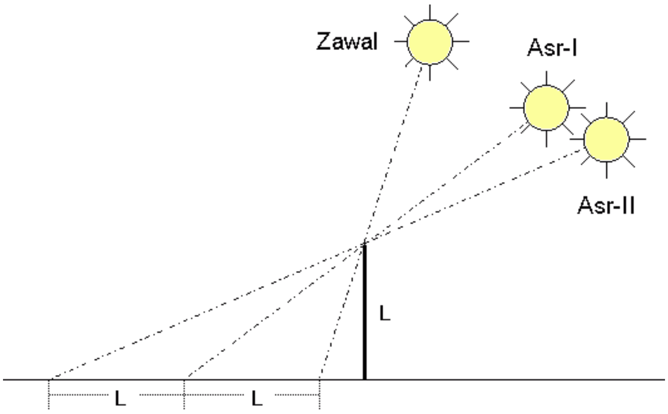

Figure 2 – Definition of Asr Time

To calculate the *asr* time, noon (zenith) shadow has to be found first. The vertical sun angle (with respect to horizon), or sun altitude at zenith will be:

$$Z = 90^\circ - |L - D| \qquad \ldots(1)$$

L is the latitude of the place, whereas D stands for declination angle of sun from celestial equator. D varies between +23º26’ (Summer Solstice) and -23º26’ (Winter Solstice) throughout the year. The absolute value is necessary for the equatorial zone. The shadow length of a 1 meter object at zenith (shortest shadow) will then be the cotangent of this angle (or tangent of its complementary):

$$|\tan(L - D)|$$

Shadow length of a 1 meter object at *asr* is hence:

$$|\tan(L - D)| + X$$

Take X=1 for *asr*-I and X=2 for *asr*-II. Vertical sun angle at *asr* can then be computed by inverse cotangent of the shadow length:

$$A = \cot^{-1}(|\cot(Z)| + X) \qquad \ldots(2)$$

$$A = \cot^{-1}(|\tan(L - D)| + X) \qquad \ldots(3)$$

Figure 3 shows *asr* altitude versus sun altitude.

The *asr* time (in hours) can then be deducted by the following general timing formula, where T is zenith (noon) local time and A is the *asr* altitude [5]:

$$T+\cos^{-1}((\sin(A)-\sin(D)*\sin(L))/(\cos(D)*\cos(L)))/15 \qquad \ldots(4)$$

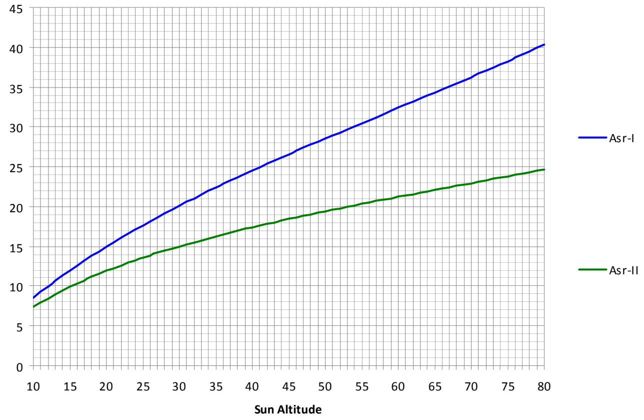

Figure 3 – Relation between Asr & Sun Altitudes

3. INFLUENCE of REFRACTION

The refraction is caused by the spherical shape of the atmosphere with a hypothetical equivalent thickness, resulting in different path lengths for the sun rays for different incident angles (Figure 4).

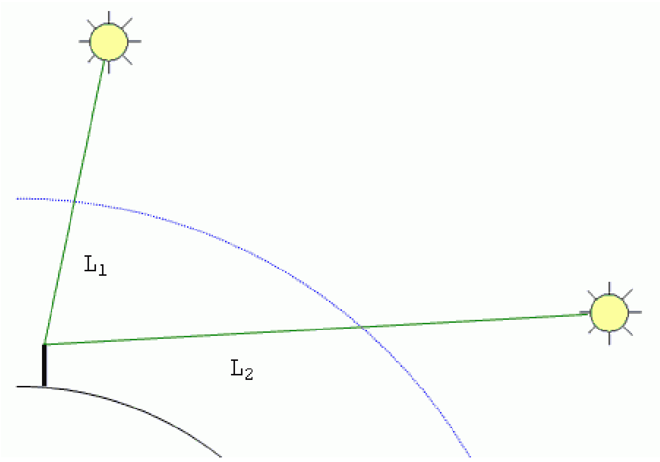

Figure 4 – Incidence and Path Length

As the sun travels away from zenith and its altitude decreases, its rays cross a more slanted path, so the total traveled path length, and in turn, the refraction angle increases. Figure 5 presents the refraction angle as a function of apparent sun altitude. Note that the refraction angle is 0.57º for zero altitude.

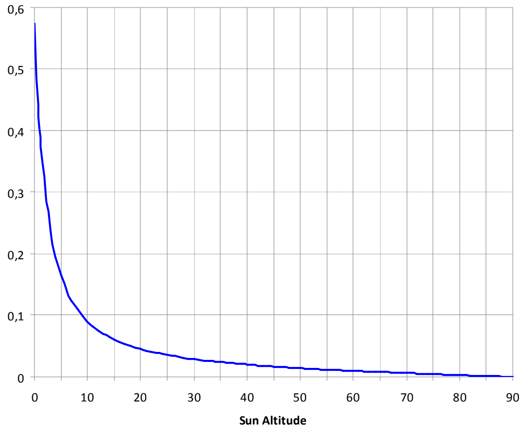

Figure 5 – Refraction versus Sun Altitude

Sæmundsson developed a simple empirical formula [7] for determining the <u>apparent</u> altitude (A') from true altitude (A):

```
A' = A+cot(A+10.3°/(A+5.11°))*0.017   ...(5)
```

Bennett suggested instead the following formula [8] for calculating the <u>true</u> altitude from the apparent altitude:

```
A = A'-cot(A'+7.31°/(A'+4.4°))/60     ...(6)
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The consequence of refraction is that the apparent altitude of the sun is always higher than its true altitude, which brings about that the shadow of any object is always smaller than it would be (without refraction). So both the noon and *asr* shadows become shorter than those calculated using the true noon angle. However, since the *asr* angle is smaller than the noon angle, the refraction at *asr* will be greater, and therefore the correct true *asr* angle will be <u>smaller</u> when we consider refraction (Figure 6). So the correct *asr* time occurs somewhat later under the effect of refraction. Below is explained the method for attaining the refraction-corrected *asr* time.

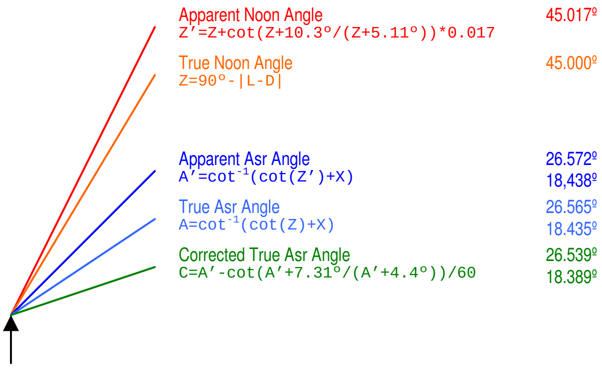

Figure 6 – Sun Vertical Angles for Asr Calculation

We first find the real shadow length of a 1 meter object. Here, we take the cotangent of the <u>apparent</u> sun altitude instead of true sun altitude. We then calculate the apparent *asr* angle and convert it back to true angle. The process is given in the following steps:

a. Find true vertical sun angle at zenith (Z) using (1).

```
Z  = 90°-|L-D|
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b. Convert it to apparent angle (Z') using (5).

```
Z' = Z+cot(Z+10.3°/(Z+5.11°))*0.017
```

c. Find apparent *asr* angle (A') using (2).

$$A' = \cot^{-1}(\cot(Z')+X)$$

d. Convert to corrected true *asr* angle (C) using (6).

```
C = A'-cot(A'+7.31°/(A'+4.4°))/60
```

e. Put this angle into (4) and get corrected *asr* time.

Figure 6 displays the definition, calculation and relation of sun vertical angles, used for *asr* timing. The numbers on the right represent an example for a true noon altitude of 45º. For each *asr* angle, the above value is for *asr*-I and the below is for *asr*-II.

We have used a spreadsheet to obtain correct *asr* altitudes for each degree of sun true noon altitude. The influence of refraction on *asr* angles, i.e. the difference (error) between the uncorrected and corrected *asr* angles (A - C) is displayed on Figure 7.

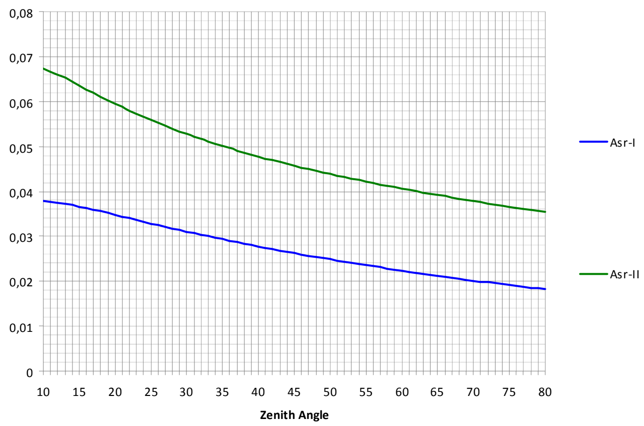

Figure 7 – Asr Angle Error due to Refraction

The error in *asr* time due to this angle difference depends upon the latitude (L) and the declination (D). It increases in winter and at high latitudes. The following table (Table 1) summarizes the *asr* time drift (delay) in seconds for different latitudes:

	Asr-I		Asr-II	
Latitude	Summer Solstice	Winter Solstice	Summer Solstice	Winter Solstice
30º	5	12	10	18
40º	6	18	12	26
50º	8	32	15	44
60º	12	85	21	117

Table 1 – Asr Time Drift due to Refraction

4. SIMPLE CORRECTION TERM

The method mentioned above to find the correct *asr* time incorporates a double conversion to and from the apparent altitude using a trigonometric function. Especially aiming an embedded implementation, we may prefer a simpler term to correct the angle error with sufficient approximation. The following single equation is offered for the calculation of corrected *asr*-I angle:

$$C1 = \cot^{-1}(|\tan(L-D)|+1) * 1.00065 - 0.0439$$

Similarly, the corrected *asr*-II angle can be approximated as:

$$C2 = \cot^{-1}(|\tan(L-D)|+2) * 1.00191 - 0.0817$$

This corrected angle (C1/C2) should then be put into equation (4) to find out correct *asr* time. Figure 8 shows the residue (approximation error in º) after using the above terms.

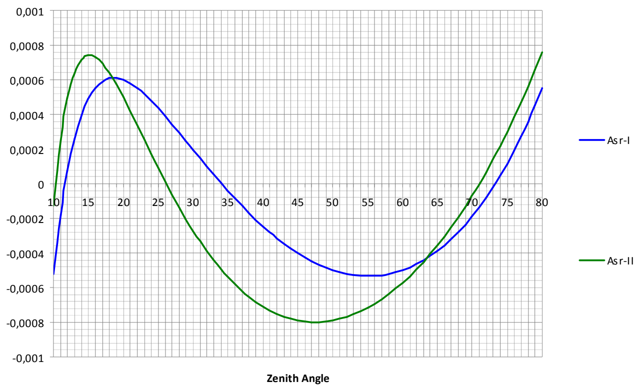

Figure 8 – Correction Term Residue

5. CONCLUSION

In this paper, we investigated the effect of atmospheric refraction on shadow length and consequently on *asr* praying timing. The refraction shortens both the noon and *asr* shadows. Since the refraction is bigger at *asr* than at noon, the true sun altitude at apparent *asr* becomes smaller. The difference turns out to be approx. 0.03º for *asr*-I and 0.05º for *asr*-II, increasing with smaller noon angles. The delay in *asr* time because of this difference is found to be less than one minute for moderate latitudes. However, especially in winter, this time drift can well exceed a minute for high latitudes. We consider that any qualified prayer time calculation software should incorporate the phenomenon depicted in this paper. To ease embedded applications, we offered a simple approximation term for the *asr* correction, which diminishes the correction error to less than 0.0006º for the first *asr* and less than 0.0008º for the second.

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